Construção e Análise de Algoritmos

lista de exercícios ll

1. A regra de Horner

   Suponha que nós queremos calcular o valor do polinômio abaixo sobre um ponto x_0

   $$P(x) \quad = \quad a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$$

   a) Mostre que a seguinte rotina, conhecida como a regra de Horner, realiza essa tarefa, deixando a resposta na variável `z`

   ```
   z = an
   for i  =  n-1 downto 0
       z  =  z * x0  +  ai
   ```

   b) How many additions and multiplications does this routine use, as a function of n? Can you find a polynomial for which an alternative method is substantially better?

2. Set sum

   Let A and B be two sets. Their sum A+B is defined to be

   $$A + B \quad = \quad \{a + b \mid a \in A, b \in B\}$$

   For example if $A = \{1, 4, 6\}$ and $B = \{2, 5\}$ then $A + B = \{3, 6, 8, 9, 11\}$.

   Give an $O(n \log n)$ algorithm whose input is two sets $A, B \subseteq \{1, ..., n\}$ and the output is set $A + B$.

   (Hint: use the FFT algorithm.)